Manifold with Boundary

정의 1 A ***topological manifold*** of dimension n ***with boundary***
is a space M with the property that $\forall p \in M, \exists$ a coordinate chart (U, ϕ) of p
s.t.ϕ is a homeo onto an open set of ($\mathbb{R}^n$ or) $\mathbb{H}^n$,
where $\mathbb{H}^n = \{x = (x_1, \cdots, x_n) \in \mathbb{R} \mid x_1 \leq 0\}$

M의 boundary를 ∂M으로 쓴다.
즉, $\partial M = \{p \in M \mid \phi(p) \in \{0\} \times \mathbb{R}^{n-1} =: \partial\mathbb{H}^n\}$

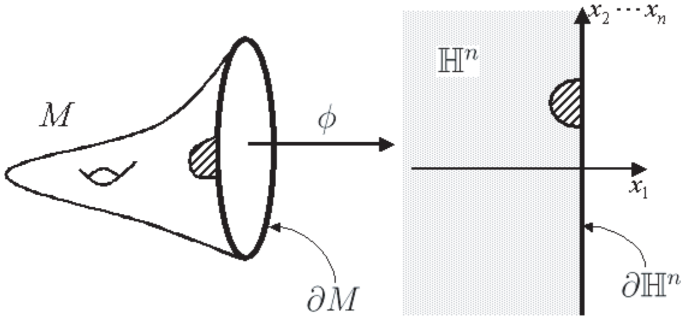

그림. Manifold with Boundary

Remark (invariance of domain)
In $\mathbb{R}^n$, 1-1 continuous image of open set is open.

위 Remark에 따르면, coordinate transition map은 interior point를 interior point로 boundary point는 boundary point로 보내므로, ∂M은 coordinate chart의 선택에 관계없이 잘 정의된다.

A ***smooth structure*** on a manifold with boundary is defined exactly same as before.
(**note.** a map defined on an open subset of $\mathbb{H}^n$ is smooth if it can be extended to a smooth map on an open set of $\mathbb{R}^n$)

• M : *n-dimensional* C^∞ manifold with boundary
$\Rightarrow \partial M$: $(n-1)$-*dimensional* C^∞ manifold without boundary